HYBRID BOUNDS FOR TWISTED L-FUNCTIONS

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ABSTRACT. The aim of this paper is to derive bounds on the critical line $\Re s = 1/2$ for L-functions attached to twists $f \otimes \chi$ of a primitive cusp form of level N and a primitive character modulo q that break convexity simultaneously in the s and the q aspect. If f has trivial nebentypus, it is shown that

$$L(f \otimes \chi, s) \ll (N|s|q)^{\varepsilon} (|s|q)^{\frac{1}{2} - \frac{1}{40}} N^{\frac{41}{40}}$$

where the implied constant depends only on $\varepsilon > 0$ and the archimedean parameter of f. To this end, two independent methods are employed to show

$$L(f \otimes \chi, s) \ll (N|s|q)^{\varepsilon} N^{\frac{5}{8}} |s|^{\frac{1}{2}} q^{\frac{3}{8}}$$

and

$$L(g, s) \ll (D|s|)^{\varepsilon} D^{\frac{2}{3}} |s|^{\frac{5}{12}}$$

for any automorphic f of level D (not necessarily a twist $f \otimes \chi$ of level $D = Nq^2$).

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