THE AVERAGE VALUE OF DIVISOR SUMS IN ARITHMETIC PROGRESSIONS

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Abstract. Let $\alpha(n)$ denote the Fourier coefficients of cusp forms or the number of divisors of n. Estimates of the type

$$\sum_{b\,(q)}{}^{*}\Big|\sum_{\substack{n\leq X\\ n\equiv b\,(q)}}\alpha(n)-\text{ main term }\Big|^2\ll_\varepsilon X^{1+\varepsilon}$$

are shown, uniformly in $q\leq X$. The methods can be extended to other arithmetic functions, e.g. the number of representations of n as a sum of two squares, or k-free numbers. As an application, sums of the type $\sum_{n\leq X}\alpha(n)\psi(n)$ for any q-periodic function ψ can be estimated non-trivially.

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2000 *Mathematics Subject Classification.* 11N37.

Key words and phrases. arithmetic functions, divisor sums, cusp forms, squarefree numbers, arithmetic progressions, square mean, character twists, Kloosterman sums.