SPECTRAL DECOMPOSITION OF SHIFTED CONVOLUTION SUMS

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ABSTRACT. Let π_1, π_2 be cuspidal automorphic representations of $\mathrm{PGL}_2(\mathbb{R})$ of conductor 1 and Hecke eigenvalues $\lambda_{\pi_{1,2}}(n)$, and let $h > 0$ be an integer. For any smooth compactly supported weight functions $W_{1,2} : \mathbb{R}^\times \to \mathbb{C}$ and any $Y > 0$ a spectral decomposition of the shifted convolution sum

$$\sum_{m \pm n = h} \frac{\lambda_{\pi_1}(|m|)\lambda_{\pi_2}(|n|)}{\sqrt{|mn|}} W_1\left(\frac{m}{Y}\right) W_2\left(\frac{n}{Y}\right)$$

is obtained. As an application, a spectral decomposition of the Dirichlet series

$$\sum_{\substack{m,n \geqslant 1 \\ m-n=h}} \frac{\lambda_{\pi_1}(m)\lambda_{\pi_2}(n)}{(m+n)^s} \left(\frac{\sqrt{mn}}{m+n}\right)^{100}$$

is proved for $\Re s > 1/2$ with polynomial growth on vertical lines in the s aspect and uniformity in the h aspect.

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