Minisymposium 26 - Mathematics in the Biosciences

Dynamic of Dirac concentrations in PDE: adaptive evolution and Turing patterns

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Several evolution equations arising in biology share the same qualitative aspect. When some parameters of the model are small, the solutions concentrate as Dirac masses that moves with a finite speed. This occurs in two examples:
(i) Adaptative evolution at the population level. This describes the selection of individuals with a trait that is better adapted to an environment shared by all the population when 'small' mutations occur.
(ii) Nonlinear parabolic equations that exhibit a Turing type instability.

We will give mathematical models of such dynamics and show that an asymptotic method allows us to describe the evolution of the 'concentration points' . Numerically, we can observ jumps in the Dirac locations, bifurcations (which lead to the cohabitation of two different populations) ot transition from dimorphism to monomorphism. In the regular regime, we obtain a canonical equation where the drift is given by a nonlinear problem.

The asymptotic method leads to evaluate the weight and position of a moving Dirac mass desribing the population. We will show that a Hamilton-Jacobi equation with constraints naturally describes this asymptotic. Some more theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.