

Minisymposium 21

Automorphic forms and their applications

Leiter des Symposiums:

PD Dr. Nils Scheithauer
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288
69120 Heidelberg, Germany

Prof. Dr. Jan H. Bruinier
Mathematisches Institut
Universität zu Köln
Weyertal 86-90
50931 Köln, Germany

Montag, 18. September

HS V, Hauptgebäude, Regina-Pacis-Weg

15:00 – 15:50	Don Zagier (<i>MPI Bonn</i>)
t.b.a.	
16:00 – 16:20	Viacheslav Nikulin (<i>Liverpool</i>)
Correspondences of a K3 surface with itself via moduli of sheaves	
16:30 – 16:50	Kathrin Bringmann (<i>Madison</i>)
Freeman Dyson's "Challenge for the Future": The mock theta functions	
17:00 – 17:20	Geoffrey Mason (<i>Santa Cruz</i>)
Ghosts and the Riemann theta functions	
17:20 – 17:50	Ulf Kühn (<i>HU Berlin</i>)
Arakelov theory on modular curves	

Dienstag, 19. September

Hörsaal 116, AVZ I, Endenicher Allee 11-13

15:00 – 15:50	Eduard Looijenga (<i>Utrecht</i>)
locally symmetric divisors in locally symmetric varieties	
16:00 – 16:20	Hironori Shiga (<i>Chiba</i>)
AGMs and some Picard modular forms	
16:30 – 16:50	Bernhard Heim (<i>MPI Bonn</i>)
Special values of automorphic L -functions	
17:00 – 17:20	Amanda Folsom (<i>MPI Bonn</i>)
Modular units	

17:30 – 17:50 **Rainer Weissauer** (*Heidelberg*)
t.b.a.

Mittwoch, 20. September

Hörsaal 116, AVZ I, Endericher Allee 11-13

15:00 – 15:50 **Gerard van der Geer** (*Amsterdam*)
Siegel modular forms and curves over finite fields

16:00 – 16:20 **Özlem Imamoglu** (*ETH Zürich*)
Zeroes of the Weierstrass p -function and hypergeometric series

16:30 – 16:50 **Shuichi Hayashida** (*Siegen*)
The Ikeda lifting and Jacobi forms of scalar index of general degree

17:00 – 17:20 **Tobias Mühlenbruch** (*Clausthal*)
Hurwitz continued fractions and Ruelle's transfer operator

17:30 – 17:50 **Nils-Peter Skoruppa** (*Siegen*)
Jacobi forms of weight 1 and applications

Vortragsauszüge

Don Zagier (MPI Bonn)
t.b.a.

(Abstrakt lag bei Redaktionsschluss noch nicht vor.)

Viacheslav Nikulin (Liverpool)
[Correspondences of a K3 surface with itself via moduli of sheaves](#)

I shall review my (some of them joint with Carlo Madonna) results about correspondences of a K3 surface with itself. Surprisingly, sometimes elements of the Picard lattice of a K3 surface permit to construct interesting 2-dimensional algebraic cycles on the product of the K3 surface with itself. E.g. see recent preprint math.AG/0605362.

Kathrin Bringmann (Madison)
[Freeman Dyson's "Challenge for the Future": The mock theta functions](#)

In his last letter to Hardy, Ramanujan defined 17 peculiar functions which are now referred to as his mock theta functions. Although these mysterious functions have been investigated by many mathematicians over the years, many of their most basic properties remain unknown. This inspired Freeman Dyson to proclaim

“The mock theta-functions give us tantalizing hints of a grand synthesis still to be discovered. Somehow it should be possible to build them into a coherent group-theoretical structure, analogous to the structure of modular forms which Hecke built around the old theta-functions of Jacobi. This remains a challenge for the future.”

Freeman Dyson, 1987, Ramanujan Centenary Conference

Here we announce a solution to Dyson's “challenge for the future” by providing the “coherent group-theoretical structure” that Dyson desired in his plenary address at the 1987 Ramanujan Centenary Conference. In joint work with Ken Ono, we show that Ramanujan's mock theta functions, as well a natural generalized infinite class of mock theta functions may be completed to obtain Maass forms, a special class of modular

forms. We then use these results to prove theorems about Dyson's partition ranks. In particular, we shall prove the 1966 Andrews-Dragonette Conjecture, whose history dates to Ramanujan's last letter to Hardy, and we shall also prove that Dyson's ranks 'explain' Ramanujan's partition congruences in an unexpected way.

Geoffrey Mason (*Santa Cruz*)
[Ghosts and the Riemann theta functions](#)

We sketch the general connection between vertex operators and automorphic forms and show that genus 2 partition functions associated with the ghost system, which is part of the bosonic string, are the corresponding Riemann theta functions.

Ulf Kühn (*HU Berlin*)
[Arakelov theory on modular curves](#)

Using the rich and well understood arithmetic of elliptic modular curves and their automorphic forms we calculate upper bounds for the Arakelov self intersection number ω^2 .

Eduard Looijenga (*Utrecht*)
[Locally symmetric divisors in locally symmetric varieties](#)

A locally symmetric variety that possesses a locally symmetric divisor is either a ball quotient or of orthogonal type (IV in the Cartan classification). In algebraic geometry, one often encounters the situation that certain algebraic objects (varieties, usually) are parameterized by a locally symmetric variety as above and that the degenerate objects are parameterized by a locally symmetric divisor. After giving some examples we shall explain why it is that the degeneracy locus tends to be like this. We also go into the question of when such a divisor is definable by an automorphic form.

Hironori Shiga (*Chiba*)
[AGMs and some Picard modular forms](#)

We present some arithmetic-geometric means (we say AGM) with three terms extending classical AGMs with two terms. We can see such an AGM as a function of two variables with normalized initial data, and this function has an expression via the Appell hypergeometric function F_1 . We can introduce these AGMs based on the theory of modular forms on the complex hyperball with respect to some Picard modular groups and the expression of some special CM-isogenies of abelian varieties.

Bernhard Heim (*MPI Bonn*)
[Special values of automorphic \$L\$ -functions](#)

The Saito-Kurokawa conjecture and a certain Hecke invariant subspace, the Maass Spezialschar, play a fundamental role in studying the Siegel three-fold. In this talk a model of the space of Siegel modular forms of the three-fold is introduced, which recovers the special values of corresponding L -functions predicted by the Gross-Prasad conjecture. The vanishing orders of these special values give a precise description of the property if a modular form is a Saito-Kurokawa lift or not. This leads to applications for Siegel modular forms of arbitrary even degree.

Amanda Folsom (*MPI Bonn*)
[Modular units](#)

In this talk I will discuss a particular family of modular units constructed using functional solutions to q -difference equations found in the work of Selberg. Arising in this way, these objects are of interest for various analytic properties and combinatorial interpretations. Dually, we exhibit fundamental algebraic roles played by these modular units, including those within the modular function fields, the modular unit groups, the cuspidal divisor class groups, class field theory, and the cyclotomic theory.

Rainer Weissauer (Heidelberg)
t.b.a.

(Abstrakt lag bei Redaktionsschluss noch nicht vor.)

Gerard van der Geer (Amsterdam)
Siegel modular forms and curves over finite fields

The cohomology of local systems on moduli spaces of curves and abelian varieties can be expressed in terms of modular forms. Moduli spaces of curves over finite fields can thus be used to obtain information about Siegel modular forms. In the lecture we will try to explain joint work with Carel Faber on Siegel modular forms of genus 2.

Özlem Imamoglu (ETH Zürich)
Zeroes of the Weierstrass p -function and hypergeometric series

Surprisingly the location of the zeroes of the Weierstrass \wp -function was not known until 1982, when Eichler and Zagier found for them a beautiful integral formula. In a joint paper with W. Duke we were able to “deuniformize” the Eichler Zagier formula and write the zeroes in terms of generalized hypergeometric series. Our formulas can be thought as the analog of classical formulas for the periods in terms of the Gauss hypergeometric series.

Shuichi Hayashida (Siegen)
The Ikeda lifting and Jacobi forms of scalar index of general degree

T. Yamazaki showed a relation between Fourier-Jacobi coefficients of Siegel Eisenstein series by using certain Hecke operators which change the index of Jacobi forms. (In case of degree 1, this Hecke operator coincides with the V_m operator introduced by Eichler-Zagier.) This relation gave a generalization of the Maass-relation for Siegel Eisenstein series. In this talk, we prove a similar relation for the Siegel cusp forms which are obtained by Ikeda lifting.

Tobias Mühlenbruch (*Clausthal*)

[Hurwitz continued fractions and Ruelle's transfer operator](#)

We report a recent development concerning the transfer operator associated to a dynamical system. This is joint work with Dieter Mayer and Fredrik Strömberg (TU Clausthal). We present the well known Hurwitz continued fractions and the associated dynamical system. We present also a Ruelle transfer operator L_β for this dynamical system. The transfer operator L_β is related to the Selberg ζ -function associated to the geodesic flow on the modular surface $SL_2(\mathbb{Z})\backslash\mathbb{H}$. Moreover, certain eigenfunctions of the transfer operator L_β have a cocycle interpretation. These cocycles are associated to Maass cusp forms using a theorem due to Bruggeman, Lewis and Zagier. Interestingly, all these connections between the stated areas in dynamical systems, ergodic theory and number theory also seem to hold for Hecke triangle groups. Finally, we present numerical calculations of the spectrum of the transfer operator for some selected Hecke triangle groups, pointing out the relation to Maass cusp forms.

Nils-Peter Skoruppa (*Siegen*)

[Jacobi forms of weight 1 and applications](#)

In the study of Siegel modular forms of critical weight and in the context of certain conjectures on product formulas for Jacobi forms the weight 1 Jacobi forms on subgroups of $SL(2, \mathbb{Z})$ play an important role. We report on the computation of dimension formulas for weight 1 Jacobi forms, some applications and open questions.