Minisymposium 9

Nichtlineare Evolutionsgleichungen und Probleme mit freiem Rand

Leiter des Symposiums:

Prof. Dr. Günther Grün
Universität Erlangen-Nürnberg
Institut für Angewandte Mathematik
Martensstr. 3
91058 Erlangen, Germany

PD Dr. Michael Winkler
Universität zu Köln

Weyertal 86-90
50931 Köln, Germany

Zahlreiche Phänomene in Natur- und Ingenieurwissenschaften lassen sich mathematisch durch Probleme mit freiem Rand oder nichtlineare Evolutionsgleichungen beschreiben.

Daraus ergeben sich vielfältige mathematische Fragestellungen. Sie reichen von Existenzaussagen für Lösungen über Regularitätsaussagen freier Ränder bis hin zu Untersuchungen zur Zeitasymptotik.

In diesem Mini-Symposium sollen neueste Resultate aus Analysis und Numerik vorgestellt werden. Im Mittelpunkt sollen dabei Ergebnisse stehen, die für die Anwendungsgebiete

- Mehrphasenströmung,
- Verbrennungsvorgänge,
- Reaktions-Diffusions-Gleichungen in Biologie und Chemie,
- Strukturbildung in Festkörpern

relevant sind.

Dienstag, 19. September

Seminarraum 1 (Raum 205), Institut für Physikalische und Theoretische Chemie
Wegelerstr. 12

15:00 – 15:50 **Lorenzo Giacomelli** *(Rom)*
A non-local degenerate parabolic system arising from strain-gradient plasticity

---

16:00 – 16:50 **Matthias Röger** *(Eindhoven)*
Sharp and diffuse interface models in phase transitions

17:00 – 17:20 **Nicolas Dirr** *(MPI Leipzig)*
Nonlinear PDEs with noise

17:30 – 17:50 **Tobias Weth** *(Giessen)*
Existence and asymptotic shape of solutions to a nonlinear Schrödinger system

---

18:00 – 18:20 **Kianhwa C. Djie** *(Aachen)*
An Upper Bound for the Waiting Time for Doubly Nonlinear Parabolic Equations

---

Mittwoch, 20. September

Seminarraum 1 (Raum 205), Institut für Physikalische und Theoretische Chemie
Wegelerstr. 12

15:00 – 15:50 **Joost Hulshof** *(Amsterdam)*
A free boundary problem for combustion with radiative effects

---

16:00 – 16:50 **Ben Schweizer** *(Basel)*
Homogenization of degenerate two-phase flow equations with a free boundary approach

---

17:00 – 17:20 **Dirk Horstmann** *(Köln)*
Analysis of some Lotka-Volterra competition model in the presence of cross-diffusion

---

17:30 – 17:50 **Adrian Muntean** *(Bremen)*
Mathematical issues concerning evolving sharp-reaction interfaces in unsaturated reactive porous materials: global well-posedness of a moving-boundary system with a kinetic condition

---

Vortragsauszüge

Lorenzo Giacomelli *(Rom)*
A non-local degenerate parabolic system arising from strain-gradient plasticity

Size effects which metals display at micron length-scales are typically described by strain-gradient dependent theories, one of which has been recently proposed by Gurtin (J. Mech. Phys. Solids, 2004): it is characterized by a free energy which depends on the density of dislocations through the Burgers tensor (the curl of the plastic displacement gradient) and which is dissipated through the plastic spin (the time derivative of the skew part of the plastic displacement gradient). Under suitable symmetry assumptions, this model leads to a non-local and degenerate parabolic system complemented with mixed boundary conditions.

I will describe the gradient flow structure of the problem and how it leads to the existence and uniqueness of solutions, as follows from a joint work with Michiel Bertsch (IAC-CNR and U. Rome "Tor Vergata"), Roberta Dal Passo and Giuseppe Tomassetti (U. Rome "Tor Vergata"). I will also discuss heuristics and open questions related both to the presence of interfaces –the boundaries of dislocation-free regions– and to two singular limits: the first one, as the ratio between the microscopic length-scale associated with the Burgers tensor and the macroscopic size of the sample vanishes, is expected to produce boundary layer effects; the second one, as the model approaches a rate-independent formulation, brings out a prototype for (possibly non-local) "infinity-curl^{2}öperators and their evolution.

Matthias Röger *(Eindhoven)*
Sharp and diffuse interface models in phase transitions

We report on recent advances in some sharp and diffuse interface models for phase transitions. In particular we study the Gamma-convergence of functionals arising in the Van der Waals-Cahn-Hilliard theory. The corresponding limit functional is given as the sum of the area and the Willmore functional. This problem was proposed as modification of a conjecture of De Giorgi.

Nicolas Dirr *(MPI Leipzig)*
Nonlinear PDEs with noise

The qualitative and quantitative analysis of solutions of nonlinear partial differential equations has proven to be a valuable tool in describing the behavior of complex materials. However in many situations the influence of thermal fluctuations cannot be neglected, in particular if the system is near an unstable equlibrium, or the long-time behavior is of interest. This leads to a perturbation of the nonlinear evolution equation by noise. We give examples where the highest order part of the PDE is not linear. Then analysis needs a combination of tools from the theory of nonlinear PDEs and from probability theory.

Tobias Weth *(Giessen)*
Existence and asymptotic shape of solutions to a nonlinear Schrödinger system

We consider the two coupled Schrödinger equations

$$-\Delta u + u = u3 + \beta v2u, \qquad -\Delta v + v = v3 + \beta u2v$$

with coupling parameter $\beta \in \mathbb{R}$. Of physical interest are bound states, i.e., positive solutions defined on the whole space and decaying exponentially at infinity. While for $\beta > 0$ all bound states are known to be radial, we show the existence of infinitely many nonradial solutions (with prescribed symmetries) in the repulsive case $\beta < -1$ in dimensions $N = 2, 3$. For the corresponding homogeneous Dirichlet problem in a ball, we also study the asymptotic shape of radial solutions in the segregation limit $\beta \to -\infty$. This is joint work with Juncheng Wei.

Kianhwa C. Djie *(Aachen)*
An Upper Bound for the Waiting Time for Doubly Nonlinear Parabolic Equations

There is a wide class of equations with the property that their solutions exhibit a waiting time phenomenon, i. e. they have a strict positive waiting time in the following sense: Let $\Omega := \mathrm{supp}(u_0) \subset \mathbb{R}^N$. Then

$$t^\star_\Omega := \sup\{t \geq 0 \mid u(x,\tau) = 0 \text{ for all } x \in \mathbb{R}^N \setminus \Omega \text{ and } \tau \in [0,t]\}$$

is called the *waiting time* for u.
We will sketch the proof for a quantitative upper bound for the waiting time for weak solutions of the doubly nonlinear parabolic equation

$$\begin{cases} (|u|^{q-2}u)_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) &= 0 \qquad \text{in } \mathbb{R}^N \times [0,\infty) \;, \\ u(x,0) &= u_0(x) \quad \text{for all } x \in \mathbb{R}^N \;, \end{cases}$$

with parameters $p \geq 2,\ 1 < q < p$ depending on the growth of the initial value u_0 near $x_0 \in \partial\Omega$. This upper bound coincides (apart from a constant factor) with the lower bound given by Giacomelli-Grün [Interfaces Free Bound. **8** No. 1 $(2006),\ 111–129$]. The technique is inspired by Chipot-Sideris [Trans. Am. Math. Soc. **288** $(1985),\ 423–427$].

Joost Hulshof *(Amsterdam)*
A free boundary problem for combustion with radiative effects

Adding radiative effects to the standard thermo-diffusive model for combustion in gaseous mixtures leads to non-trivial bifurcation diagrams for travelling wave solutions and radial flame ball solutions if one varies the parameters. I plan to concentrate my talk on the changes of stability in these diagrams.

Ben Schweizer *(Basel)*
Homogenization of degenerate two-phase flow equations with a free boundary approach

We consider the one-dimensional degenerate two-phase flow equations as a model for water-drive in oil recovery. The effect of oil trapping is observed in strongly heterogeneous materials with large variations in the permeabilities and in the capillary pressure curves. In such materials, a vanishing oil saturation may appear at interior interfaces and inhibit the oil recovery. We introduce a free boundary problem that separates a critical region with vanishing permeabilities from a strictly parabolic region and give a rigorous derivation of the effective conservation law.

Dirk Horstmann *(Köln)*
Analysis of some Lotka-Volterra competition model in the presence of cross-diffusion

In this talk we analyze the effect of cross-diffusion terms on some Lotka-Volterra models. Some aspects that will be discussed is the existence and the nonexistence of nontrivial steady state solution. Furthermore, we will look for traveling wave solutions for some explicit systems. In particular, we will analyze the following cross-diffusion system:

$$\begin{aligned} u_t &= \Delta u + e\Delta v + u^p(1-u-cv) \\ v_t &= d\Delta v + f\Delta u + v^q(a-bu-v) \end{aligned}$$

We will establish some existence results for traveling wave solutions of this system and we will analyze their stability properties.

Adrian Muntean *(Bremen)*
Mathematical issues concerning evolving sharp-reaction interfaces in unsaturated reactive porous materials: global well-posedness of a moving-boundary system with a kinetic condition

A variety of reaction-diffusion scenarios taking place in unsaturated reactive porous materials involve the formation and propagation of moving-sharp interfaces, where fast chemical reactions are assumed to happen. When spatially separated reactants meet, the separation boundary may be assumed as sharp provided that the characteristic time scale of reaction is much smaller than that of transport. We illustrate this situation by means of a non-equilibrium one-dimensional moving-boundary system with a kinetic condition modeling the driving force. We employ this formulatoion in order to study the evolution of the carbonation reaction in concrete-based materials. The model relies on the idea that carbonation might be considered to be localized on a sharp interface. It consists of a set of semi-linear mass-balance equations coupled with a non-linear ordinary differential equation, which accounts for the motion of the interface. We refer to this differential equation driving the sharp-reaction interface into the material as the *kinetic (non-equilibrium) condition*. The model equations are non-linearly coupled by the *a priori* unknown position of the moving-reaction interface and non-linearities in the production terms. We show that if the model parameters are selected such that local strict positive weak solutions to our model exist, then the kinetic condition can be suitably used to extend these local solutions globally in time. Finally, we present numerical predictions of penetration depths and show expected qualitative behaviors and feasibility of both the model and numerical approach. This is joint work with Michael Böhm (University of Bremen) supported by the DFG through the special priority program SPP 1122 *Prediction of the Course of Physicochemical Damage Processes involving Mineral Materials*.